Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE
AMERICAN
MATHEMATICAL MONTHLY.

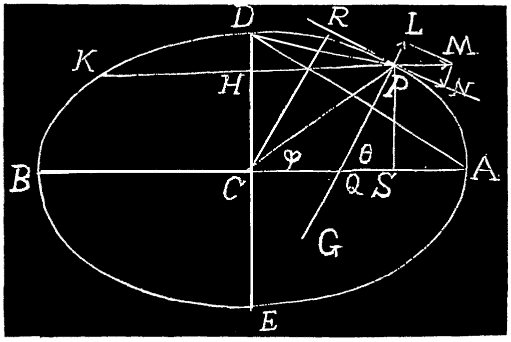
Entered at the Post-office at Springfield, Missouri, as second-class matter.

VOL. IX. FEBRUARY, 1902. No. 2.

GRAVITY, TRUE AND APPARENT.

By G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.

Let P be any point on the earth's surface, $CA=CB=a$, the earth's equatorial radius, $CD=CE=b$, the earth's polar radius, (x, y) the coördinates of P, $CP=r$, $\angle PQA=\theta=$latitude of P, $\angle PCA=\varphi$, mass of particle at P=unity, $f=$ centrifugal force of particle at P, $e=$earth's eccentricity, $g'=$true gravity in any latitude, $g=$apparant gravity in any latitude, $G'=$true gravity at the equator, $G=$apparant gravity at the equator, $l'=$length of second's pendulum for true gravity in any latitude, $l=$length of second's pendulum for apparent gravity in any latitude, $L'=$length of second's pendulum for true gravity at the equator, $L=$length of second's pendulum for apparent gravity at the equator, $T=$number of seconds in a siderial day.



Then $HP=r\cos\varphi=x$, $CH=r\sin\varphi=y$, $HP=CQ+QS$, $CQ=e^2x=e^2r\cos\varphi$.

$$PQ=\frac{b}{a}\sqrt{(a^2-e^2x^2)}=\sqrt{(b^4/a^2+e^2y^2)}=\frac{b}{a}\sqrt{(a^2-e^2r^2\cos^2\varphi)}$$

$$=\sqrt{(b^4/a^2+e^2r^2\sin^2\varphi)},\ QS=PQ\cos\theta.$$

$$\therefore\ r\cos\varphi=e^2r\cos\varphi+\frac{b\cos\theta}{a}\sqrt{(a^2-e^2r^2\cos^2\varphi.)}$$

$$\therefore\ r\cos\varphi=\frac{a^2\cos\theta}{\sqrt{(b^2+a^2e^2\cos^2\theta)}}=\frac{a\cos\theta}{\sqrt{(1-e^2\sin^2\theta)}}.$$

$$PS=CH=r\sin\varphi=PQ\sin\theta=\sin\theta\sqrt{(b^4/a^2+e^2r^2\sin^2\varphi)}.$$

$$\therefore\ r\sin\varphi=\frac{b^2\sin\theta}{a\sqrt{(1-e^2\sin^2\theta)}}=\frac{b\sqrt{(1-e^2)}\sin\theta}{\sqrt{(1-e^2\sin^2\theta)}}.\quad f=v^2/HP=\frac{4\pi^2HP}{T^2}.$$

The component of f along PQ is $PL=f\cos\theta$.

$$\therefore\ f\cos\theta=\frac{4\pi^2HP\cos\theta}{T^2}=\frac{4a\pi^2\cos^2\theta}{T^2\sqrt{(1-e^2\sin^2\theta)}}.$$

According to the spheroid of Listing of Göttingen, 1878, $a=6377377$ meters$=20923536$ feet, $b=6355270$ meters$=20851005$ feet.

$$\therefore\ e^2=\frac{a^2-b^2}{a^2}=.00692.\quad \text{But } T=86164 \text{ seconds.}$$

$$\therefore\ f\cos\theta=\frac{.1112553\cos^2\theta}{\sqrt{(1-e^2\sin^2\theta)}}=.1112553[1-(1-\tfrac{1}{2}e^2)\sin^2\theta] \text{ nearly,}$$

$$=.1112553(1-.99654\sin^2\theta) \text{ feet per second.}$$

According to Newcomb and Holden the true gravity is given by $g'=G'(1+.00173\sin^2\theta)$ for any latitude.

Now $g'-f\cos\theta=g$, also $G'=32.2015235$.

$\therefore\ g=32.2015235(1+.00173\sin^2\theta)-.1112553(1-.99654\sin^2\theta)$

$=32.0902682(1+.00519\sin^2\theta)=G(1+.00519\sin^2\theta)$.

$.00173=\frac{1}{4}e^2,\ .00519=\frac{3}{4}e^2$.

$\therefore\ g'=G'(1+\frac{1}{4}e^2\sin^2\theta),\ g=G(1+\frac{3}{4}e^2\sin^2\theta)$.

In all previous writings these formulae are given in the form $g=g_0(1+c\sin^2\theta)$ where c is a constant. Whether this constant is a function of any dimension of the spheroid or whether it has any connection with the spheroid is not stated. I here express both constants in terms of the eccentricity. Dr. Mendenhall, the greatest American authority on gravity, has told me that these constants were never before so expressed.

$$l'=g'/\pi^2=\frac{32.2015235}{9.8696044}(1+\tfrac{1}{4}e^2\sin^2\theta)=3.2626965(1+\tfrac{1}{4}e^2\sin^2\theta)\text{ feet,}$$

$$=39.1523578(1+\tfrac{1}{4}e^2\sin^2\theta)\text{ inches}=994.45(1+\tfrac{1}{4}e^2\sin^2\theta)mm.$$

$$\therefore\ l'=L'(1+\tfrac{1}{4}e^2\sin^2\theta).$$

$$l=g/\pi^2=\frac{32.0902682}{9.8696044}(1+\tfrac{3}{4}e^2\sin^2\theta)=3.251424(1+\tfrac{3}{4}e^2\sin^2\theta)\text{ feet}$$

$$=39.017088(1+\tfrac{3}{4}e^2\sin^2\theta)\text{ inches}=991.01(1+\tfrac{3}{4}e^2\sin^2\theta)mm.$$

$$\therefore\ l=L(1+\tfrac{3}{4}e^2\sin^2\theta).$$

To find the time of revolution of the earth on its axis so that centrifugal force may equal gravity.

First, at the equator.

Let A be the bob of a conical pendulum, $DC=b=$height of cone generated by this pendulum. The formula for time of revolution of this pendulum is

$$t=2\pi\sqrt{\frac{b}{G}}=2\pi\sqrt{\frac{12b}{\pi^2L'}}=2\sqrt{\frac{12b}{L'}}$$

where L' is given in inches, and b in feet.

$$\therefore\ t=2\sqrt{\frac{20851005}{3.2626965}}=5056\text{ seconds.}$$

At present the earth revolves once 86164 seconds. 86164÷5056=17.04 times as fast as at present.

Second, in latitude 30°.

Let P be the bob of the pendulum.

Then $t=2\pi\sqrt{\frac{DH}{g'}}=2\sqrt{\frac{12DH}{l'}}$, l' in inches, DH in feet.

$$DH=b-CH=b-r\sin\varphi=b-\frac{b\sqrt{(1-e^2)}\sin\theta}{\sqrt{(1-e^2\sin^2\theta)}}=b\left(1-\sqrt{\frac{1-e^2}{4-e^2}}\right),\text{ since }\theta=30^\circ.$$

$$\therefore\ DH=.5013b=10452608.8065\text{ feet.}$$

$$l'=39.1523578(1+\tfrac{1}{16}e^2)=39.16929336\text{ inches.}$$

$$\therefore\ t=2\sqrt{\frac{10452608.8065}{3.26410778}}=3579\text{ seconds.}$$

86164÷3579=24.075 times as fast as at present.

∴ One revolution per hour.

By this method we can find the time of revolution for any latitude. It is

$$t=4\sqrt{\frac{3b}{L'}}\left\{\frac{\left(1-\frac{\sqrt{(1-e^2)}\sin\theta}{\sqrt{(1-e^2\sin^2\theta)}}\right.}{(1+\frac{1}{4}e^2\sin^2\theta)}\right\}^{\frac{1}{2}}$$

where b is taken in feet, and L' in inches.

The following table gives gravity and the length of the second's pendulum for every five degrees under existing conditions.

DEGREES	GRAVITY		SECOND'S PENDULUM	
	POUNDALS	DYNES	INCHES	*mm*
0°	32.0002682	978.09	39.017088	991.01
5°	32.0915197	978.13	39.018610	691.05
10°	32.0952903	978.25	39.023194	991.17
15°	32.1014356	978.43	39.030666	991.36
20°	32.1097470	978.69	39.040771	991.62
25°	32.1200159	979.00	39.053257	991.93
30°	32.1319053	979.36	39.067713	992.30
35°	32.1450623	979.76	39.083710	992.71
40°	32.1590697	980.19	39.100741	993.14
45°	32.1735424	980.63	39.118337	993.59
50°	32.1880023	981.07	39.135918	994.03
55°	32.2020226	981.50	39.152965	994.47
60°	32.2151796	981.90	39.168962	994.87
65°	32.2270722	982.26	39.183422	995.24
70°	32.2373379	982.57	39.195903	995.56
75°	32.2456621	982.83	39.206024	995.81
80°	32.2517955	983.02	39.213482	996.00
85°	32.2555651	983.13	39.218065	996.12
90°	32.2568167	983.17	39.219587	996.16

A PROBLEM CONNECTED WITH MERSENNE'S NUMBERS.

By HARRY S. VANDIVER, Bala, Pa.

In Sir W. R. Ball's *Recreations and Problems* (London, 1809), page 33, I find the following:

"A curious proposition which comes from China, and which I believe appears here in print for the first time, is *that* $(2^n-2)/n$ *is an integer if* n *is a prime and is not an integer if* n *is not a prime.* The first of these statements is at once demonstrable; but I have not succeeded in proving the second part of the propo-